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Intersection rules, dynamics and symmetries

François Englert

Service de Physique Théorique, Université Libre de Bruxelles
Campus Plaine, C.P.225, Boulevard du Triomphe, B-1050 Bruxelles, Belgium
E-mail: `fenglert@ulb.ac.be`

Laurent Houart*

Service de Physique Théorique et Mathématique, Université Libre de Bruxelles
Campus Plaine C.P. 231, Boulevard du Triomphe, B-1050 Bruxelles, Belgium
E-mail: `lhouart@ulb.ac.be`

Peter West

Department of Mathematics, King's College London
Strand, WC2R 2LS London UK
E-mail: `pwest@mth.kcl.ac.uk`

Abstract: We consider theories containing gravity, at most one dilaton and form field strengths. We show that the existence of particular BPS solutions of intersecting extremal closed branes select the theories, which upon dimensional reduction to three dimensions possess a simple simply laced Lie group symmetry $\mathcal{G}$. Furthermore these theories can be fully reconstructed from the dynamics of such branes and of their openings. Amongst such theories are the effective actions of the bosonic sector of M-theory and of the bosonic string. The BPS intersecting brane solutions form representations of a subgroup of the group of Weyl reflections and outer automorphisms of the triple Kac-Moody extension $\mathcal{G}^{+++}$ of the $\mathcal{G}$ algebra, which cannot be embedded in the overextended Kac-Moody subalgebra $\mathcal{G}^{++}$ characterising the cosmological Kasner solutions.

Keywords: Bosonic Strings, M-Theory, p-branes, Space-Time Symmetries.

*Research Associate F.N.R.S.

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1. Introduction and conclusion

There is a widespread belief that a consistent quantum theory of gravity and matter should emerge from some non perturbative M-theory generalisation of superstrings admitting eleven dimensional supergravity as its low energy effective action. Upon compactification, its bosonic sector displays remarkable symmetries which are often viewed as the signature of an underlying supersymmetry and are assumed to be symmetries of the full would-be non perturbative string theory. Supersymmetry controls the existence of zero binding energy bound states and leads to one of the most celebrated result of the string theory approach to quantum gravity, namely the correct counting of states of some extremal and nearly extremal black holes built from extremal intersecting branes.

In this paper, we show that the existence of such zero binding energy states in classical general relativity is not a privilege of M-theory effective actions. We find a large class of theories, all of which contain gravity, sharing these features. Interestingly, these theories include the low energy effective action of the bosonic string.

We consider generic theories in D dimensions including gravity, one dilaton and form field strengths of arbitrary degree and arbitrary couplings to the dilaton. These theories admit solutions describing parallel extremal branes, i.e. charged solitonic extended objects with no force between them. Under precise circumstances there exists configurations of closed extremal branes intersecting orthogonally in a configuration with zero binding energy. In this paper we will use the BPS terminology for these configurations.[1] We will review the general intersection rules [1] which determine for which theories such BPS configurations exist. This will lead to constraints on the dimension D, the admissible field strengths and their dilaton couplings. In particular cases the existence of such BPS configurations will allow a complete determination of the D-dimensional theory, including its Chern-Simons terms, and will imply that the dimensionally reduced theory possesses a Lie algebra symmetry.

[1] BPS is used here in the original sense without referring to any supersymmetry property.

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This happens in the following way. We first show that the existence of BPS intersecting solutions with *one* common direction between electric and magnetic p-branes ($p \geq 1$) pairs, stemming from the form field strength present in the theory, is a necessary condition to enhance the symmetry of the dimensionally reduced theory to three space-time dimensions, from the torus deformation group $\mathrm{GL}(D-3)$ to a simply laced Lie group $\mathcal{G}$ of the same rank. We then consider the possible appearance of open branes terminating on closed ones. This possibility arises when the intersection rules giving BPS configurations are such that the number of dimensions on which two extremal branes intersect is equal to the boundary dimension of one of them. It is known that in some theories, e.g. the theories describing the different phases of M-theory, type IIA [2], type IIB [3] and 11 supergravity [4], consistency of openings is ensured by the presence of Chern-Simons terms in the theory [5]–[7]. Here we will reverse the logic. We start with a theory which admits certain branes and by analysing the BPS configurations and demanding that all possible openings be realised we find that the theory must possess Chern-Simons terms. These terms may in turn imply the introduction of new form field strengths . This constructive process can be iterated until all possible openings are consistent with the Chern-Simons terms they require.

In this construction the presence of BPS configurations will imply the existence of a simply laced symmetry group $\mathcal{G}$ in the theory dimensionally reduced to three dimensions. It is then inevitable that the theories we construct in this way contain their oxidation end-points [8]. These are the theories defined at the highest available space-time dimension D which upon dimensional reduction to three dimensions are expressible as non linear realisations by scalar fields in a coset space $\mathcal{G}/\mathcal{H}$ where $\mathcal{H}$ is the maximal compact subgroup of $\mathcal{G}$.

We have thus uncovered a relation between brane physics and the presence of symmetry. All these maximally oxidised theories do possess the same type of BPS configurations. While some of these theories, such as the bosonic sector of eleven dimensional supergravity, can be extended to possess supersymmetry which protects the BPS conditions against quantum corrections, a number of other theories do not. However, all the theories share very similar symmetries which may have important consequences at the quantum level. It has been proposed that there is a huge algebraic structure characterising eleven-dimensional supergravity which is the very extended Kac-Moody algebra of the E_8 algebra, called E_{11} (or E_8^{+++}) [9]. The same very extended algebra occurs in IIA [9] and IIB supergravity [10]. Furthermore, there has indeed been evidence that such a structure is realised, not only for supergravity, but for its proposed counterpart in the effective action of the bosonic string, namely k_{27} or D_{24}^{+++} [9] and also for pure D dimensional gravity where the proposed underlying symmetry algebra is thought to be A_{D-3}^{+++} [11]. The notion of very extended algebras was introduced, and some of their properties were discussed in reference [12]. It has been suggested that all the maximally oxidised theories, possess the very extension $\mathcal{G}^{+++}$ of the simple Lie algebra $\mathcal{G}$ [13]. The relation between branes and symmetries found in this paper lends further support for these ideas. We indeed show that the BPS solutions of the oxidised theory of a simply laced group $\mathcal{G}$ form representations of a subgroup of the Weyl transformations of the algebra $\mathcal{G}^{+++}$.

The common BPS dynamical origin of such hidden symmetries in maximally oxidised theories suggests that very extended Kac-Moody algebras generalise the role played by

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supersymmetry in some of these theories. This is in line with the suggestion that fermionic strings and supersymmetry originate in the fermionic subspaces of the compactified bosonic string to 10-dimensional space-time [14]. We recall that these subspaces accommodate all D-branes of the 10-dimensional fermionic string theories [15], yet they provide no hint about the non-perturbative origin of the $NS5$-brane out of the bosonic string. The BPS configuration of the electric string and its 21-magnetic dual in the bosonic string effective action considered here in section 2 allows to view these objects, upon such compactification, as parents of the fermionic BPS string and its $NS5$-dual.

The paper is organised as follows. In sections 2 and 3, we review and illustrate respectively the intersection rules for extremal closed branes and the conditions allowing their openings. In section 4, we relate the possible emergence of symmetry in dimensional reduction to the intersection rules. We show that the existence of BPS configurations consisting of extremal electric p-branes and of their magnetic duals intersecting along one common dimension constitute a necessary condition for having a maximally oxidised theory of a simple simply laced Lie group $\mathcal{G}$. In section 5, we use the opening of branes to generate Chern-Simons terms and effectively reconstruct *all* such theories. In section 6, we relate the existence of the BPS solutions to the group of of Weyl reflections and outer automorphisms of the triple Kac-Moody extensions $\mathcal{G}^{+++}$ of the algebra of $\mathcal{G}$.

2. Intersection rules

In this section we will review along the line of ref. [1] the general rule determining how extremal branes can intersect orthogonally in a configuration with zero binding energy.

We begin with a generic theory in D dimensions which includes gravity, a dilaton and $\mathcal{M}$ field strengths of arbitrary form degree n_I with $n_I \leq D/2$ and arbitrary couplings to the dilaton a_I. If $n_I > D/2$ we replace the field strength in the action by its dual defined by

$$\sqrt{-g}e^{a\phi}F^{\mu_1\ldots\mu_n} = \frac{1}{(D-n)!}\epsilon^{\mu_1\ldots\mu_n\nu_1\ldots\nu_{D-n}}\tilde{F}_{\nu_1\ldots\nu_{D-n}}\,. \tag{2.1}$$

The action is

$$S = \frac{1}{16\pi G_N^{(D)}}\int d^Dx\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \sum_I \frac{1}{2n_I!}e^{a_I\phi}F_{n_I}^2\right), \quad I = 1\ldots\mathcal{M}\,, \tag{2.2}$$

where we have not included possible Chern-Simons terms. It can be shown that they have no effect on the zero binding energy solutions considered here although such terms will be important in the following sections. This omission apart, the above action describes the bosonic sectors of all supergravity theories, the prototype examples being eleven dimensional supergravity [4], and the IIA [2] and IIB [3] supergravities. In this paper, we will also consider theories which do not possess supersymmetry but which are still described by eq. (2.2).

The zero binding energy configurations of closed extremal branes intersecting orthognally are obtained by first specialising to metrics of the following diagonal form,

$$ds^2 = -B^2dt^2 + \sum_{i=1}^{p} C_{(i)}^2(dy^i)^2 + \sum_{a=2}^{D-p} G^2(dx^a)^2\,, \tag{2.3}$$

where y^i are compact coordinates and the functions B, $C_{(i)}$, G depend only on the overall transverse coordinates x^a and we allow for multicenter solutions. We choose p so that all branes are wrapped in the compact dimensions and that no compact dimension is transverse to all branes. The overall transverse space is non-compact and has dimension $D-p-1$. We recall that for each brane of dimension q present in the solution with q less than p, we have to take, in the compact directions transverse to the brane, a "lattice" of such q-branes and then to average over them.

For the n-form field strengths we may choose two different ansätze,

$$\text{Electric}: F_{ti_1\dots i_{q_A}a} = \epsilon_{i_1\dots i_{q_A}} \partial_a E_A\,, \tag{2.4}$$

$$\text{Magnetic}: \tilde{F}_{ti_1\dots i_{q_A}a} = \epsilon_{i_1\dots i_{q_A}} \partial_a \tilde{E}_A\,. \tag{2.5}$$

The space-time charges are thus respectively defined by

$$Q_A^{\text{el}} \propto \int *F_{q_A+2}\,, \qquad Q_A^{\text{mag}} \propto \int F_{D-q_A-2}\,. \tag{2.6}$$

Here, the dual $*F$ is defined by eq. (2.1) and $A = 1\dots\mathcal{N}$, where $\mathcal{N}$ is the total number of electric and magnetic distinct non-parallel branes. This number can of course exceed the number of different n-forms.

We recall briefly how in reference [1] solutions of the equations of motion of the general action eq. (2.2) describing zero binding energy configurations between $\mathcal{N}$ distinct non-parallel extremal branes were found. Two ansätze where made in order to ensure the no-force condition between the constituent branes. The first one, which considerably simplifies the equations of motion, is

$$BC_1\dots C_p G^{D-p-3} = 1\,. \tag{2.7}$$

The second expresses that one independent harmonic function is associated to each brane, and that the solution is completely characterised by $\mathcal{N}$ harmonic functions H_A with $E_A \propto H_A^{-1}$ and $A = 1\dots\mathcal{N}$, where we have dropped the tilde distinguishing electric and magnetic ansätze.

The Einstein equations for the diagonal $R^a{}_a$ components and the dilaton equations yield the following expressions for an overall transverse space[2] $D-p>3$,

$$ds^2 = -\prod_A H_A^{-2\frac{D-q_A-3}{\Delta_A}} dt^2 + \sum_{i=1}^{p} \prod_A H_A^{-2\frac{\delta_A^{(i)}}{\Delta_A}} (dy^i)^2 + \sum_{a=2}^{D-p} \prod_A H_A^{2\frac{q_A+1}{\Delta_A}} (dx^a)^2\,, \tag{2.8}$$

$$e^{\phi} = \prod_A H_A^{\varepsilon_A a_A \frac{D-2}{\Delta_A}}\,, \tag{2.9}$$

with

$$H_A = 1 + \sum_k \frac{c_A Q_{A,k}}{|x^a - x_k^a|^{D-p-3}}\,, \tag{2.10}$$

[2]For $D-p \leq 3$, space is not asymptotically flat but the solution is still characterised by harmonic functions and the intersection rule equations eq. (2.11) below still apply.

where the $Q_{A,k}$ are the charges of the branes located at x_k^a and the c_A are constants. $\Delta_A = (q_A+1)(D-q_A-3)+\frac{1}{2}a_A^2(D-2)$, $\varepsilon_A = +(-)$ for electrically (magnetically) charged branes, and $\delta_A^{(i)} = D - q_A - 3$ or $-(q_A + 1)$ depending on whether y_i is parallel or perpendicular to the q_A-brane. In order to build up the metric, we thus include, for each q_A-brane in the configuration, a factor $H_A^{-2\frac{D-q_A-3}{\Delta_A}}$ in front of each coordinate (including the time coordinate) longitudinal to the brane, and a factor of $H_A^{2\frac{q_A+1}{\Delta_A}}$ in front of each transverse coordinate. This is in agreement with the harmonic superposition rule formulated first in reference [16] in the particular context of M-theory. The solution above eq. (2.8) is asymptotically flat and the total mass of such a configuration is, as expected, the sum of the masses of each constituent brane, which are equal to the charges: $M = \sum M_A = \sum Q_A$.

In this model independent way of deriving zero binding energy configurations we get an important bonus namely the *intersection rule equations* [1]. We did not yet use the $R^a{}_b$ off-diagonal components of the Einstein equations and these reduce to a set of algebraic conditions. These are the intersection rules equations which yield, for each pair (A, B) of distinct q-branes of dimensions (q_A, q_B), the number of dimensions $\bar{q}$ on which they intersect in terms of the total number of space-time dimensions D and of the field strength couplings to the dilaton. They read

$$\bar{q} + 1 = \frac{(q_A + 1)(q_B + 1)}{D - 2} - \frac{1}{2}\varepsilon_A a_A \varepsilon_B a_B \,. \tag{2.11}$$

Thus, in every theory whose action has the form shown in eq. (2.2), a zero binding energy configuration ($M = \sum M_A = \sum Q_A$) between any chosen set of extremal branes of the theory exists provided the pairwise conditions eq. (2.11) yield integer[3] $\bar{q}$ in the range $(-1 \le \bar{q} \le q_A, q_B)$. The configuration is then described by the metric eq. (2.8), the dilaton eq. (2.9), and the field strength expressed in terms of the H_A given in eq. (2.10).

It is interesting to point out some properties of the pairwise intersection rule equations. Consider a theory containing a form F_{n_A} and hence extremal electric q_A^e-brane and magnetic q_A^m-brane ($q_A^m = D - q_A^e - 4$) solutions and a form F_{n_B} and hence corresponding q_B^e and q_B^m-branes. It is easy to see from eq. (2.11) that if one of the four possible intersections between a q_A-brane and a q_B-brane has integer $\bar{q}$, then the three other intersections have also integer dimensions. We have indeed

$$\begin{aligned}
\bar{q}^{(e_A,e_B)} &= q_A^e - \bar{q}^{(e_A,m_B)} - 1\,,\\
\bar{q}^{(m_A,m_B)} &= D - q_B^e - \bar{q}^{(e_A,m_B)} - 5\,,\\
\bar{q}^{(m_A,e_B)} &= q_B^e - q_A^e + \bar{q}^{(e_A,m_B)}\,,
\end{aligned} \tag{2.12}$$

where in $\bar{q}$ the first superscript labels the electric or magnetic nature of the q_A-brane and the second that of the q_B-brane. In particular, putting $A = B$, the relations eq. (2.12) are valid for the branes arising from a single form.

The intersection rules reviewed here were applied to the different "phases" of M-theory (11D supergravity, type IIA and type IIB theories) in [1]. They gave back the well-known

[3] The case $\bar{q} = -1$ is relevant and can be interpreted in terms of instantons in the Euclidean. In that case the time coordinate need not be longitudinal to all branes.

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zero binding energy configurations preserving some supersymmetries. These brane configurations were originally derived from supersymmetry and duality arguments (see for instance [17] and reference therein).[4] The generality of the intersection rules allows 'BPS' configurations of extremal branes with vanishing binding energy in the broader context where supersymmetry is not required.

We now illustrate by one important example the emergence of such BPS states in non-supersymmetric theories. We take the effective action eq. (2.2) of the bosonic string (omitting the tachyon field) generalised to D dimensions. The corresponding string theory is consistent at the quantum level only for $D = 26$ but in the present context nothing prevent us to extend the action eq. (2.2) to any dimension D, keeping only a 3-form[5] denoted in what follows H_3. One has

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12} e^{a_3 \phi} H_3^2 \right), \tag{2.13}$$

with the dilaton coupling a_3 given by

$$a_3 = \sqrt{\frac{8}{D-2}}. \tag{2.14}$$

The 3-form H_3 gives rise to an extremal electric 1-brane describing the fundamental string, traditionally denoted $1F$, and to its dual $D-5$ magnetic brane that we denote $NS(D-5)$. We apply the intersection rules to seek for possible BPS states of intersecting extremal branes in the model. Using eq. (2.14) in eq. (2.11) we immediately see that a BPS solution exists for the three possible pairwise intersecting configurations. We have

$$1F \cap 1F = -1 \qquad 1F \cap NS(D-5) = 1 \qquad NS(D-5) \cap NS(D-5) = D-7\,, \tag{2.15}$$

where we used the notation $q_A \cap q_B = \bar{q}$. The corresponding BPS solutions are given by eq. (2.8) and eq. (2.9) with dilaton coupling eq. (2.14). From the pairwise rules we can then construct BPS configurations with more than two branes. Applying these results to the bosonic string theory, namely taking $D = 26$, we can build BPS configurations between various $1F$ and $NS21$.

3. The opening of branes

In this section we analyse the breaking of closed extremal branes into open branes terminating on closed ones. We consider the BPS configurations given by eq. (2.11) in the special case when $\bar{q}$ has the same dimension as the potential boundary of one of the two constituent branes, i.e $q_A - 1$ or $q_B - 1$, and study its possible opening. Such openings require the addition of Chern-Simons terms to the action eq. (2.2) and may enlarge the brane content of the theory. We shall see in section 5 that, under some conditions, such

[4] In [18] a derivation of the intersection rules not based on supersymmetry has been given for the branes of M-theory using brane probes in brane backgrounds.

[5] Since we have chosen $n_I \leq D/2$ we have here $D \geq 6$.

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openings fully determine the theory and relate brane dynamics to the existence of a symmetry. The presentation given in this section is a generalisation of the one performed in the context of M-theory [5]–[7].

Let us review how extended objects carrying a conserved charge can be opened. The main obstacle towards opening of branes is charge conservation. Generically, the charge density of a q-brane is measured by performing an integral of the relevant field strength on a $(D-q-2)$-dimensional sphere S^{D-q-2} surrounding the brane in its transverse space,

$$Q_q \propto \int_{S^{D-q-2}} *F_{q+2}\,. \tag{3.1}$$

If the brane is open, we can slide the S^{D-q-2} off the loose end and shrink it to zero size. This would imply the vanishing of the charge and hence a violation of charge conservation. This conclusion is avoided if, in the above process, the S^{D-q-2} necessarily goes through a region in which the equation

$$d * F_{q+2} = 0 \tag{3.2}$$

no longer holds. We shall see that this is the case when the open brane ends on some other one.

In the framework of M-theory the source terms needed in eq. (3.2) to ensure charge conservation for the open branes originate from two requirements whose interplay leads to a consistent picture. On the one hand there are space-time Chern-Simons type terms in supergravity which allow for charge conservation for well defined pairing of open and 'host' branes [6]. On the other hand the world-volume effective actions [5] for the branes of M-theory relate world-volume fields and pull-backs of space-time fields, and gauge invariance [19] for open branes ending on the 'host' brane implies that the end of the open branes acts as a source for the world-volume field living on the closed 'host' brane.

In reference [7] a systematic study in M-theory of all the zero binding energy configurations eq. (2.11) corresponding to $\bar{q} = q_A - 1$ (with $q_A \leq q_B$) was performed. It was shown that in all cases it was possible to open the q_A-brane along its intersection with the q_B-brane. The crucial ingredient was the presence of the appropriate Chern-Simons terms in the supergravity lagrangians for each case.

Here we propose to reverse the logic. Starting with a lagrangian of type eq. (2.2) and having zero binding energy configurations between branes we will ask that, if $\bar{q} = q_A - 1$, the corresponding q_A-brane open on the q_B-brane. This will *determine* the form of the Chern-Simons terms one has to add to eq. (2.2) and will also in some cases require the *introduction of new field strength forms* F_{n_I}. One then proceeds iteratively.

We illustrate the role of the Chern-Simons term (see also [6, 7]) by taking as example a theory with only one n-form F_{q^e+2} and dilaton coupling in eq. (2.2) such that the intersection rule between the electric q^e-brane and the magnetic q^m-brane ($q^m = D - q^e - 4$) is $\bar{q} = q^e - 1$.

We modify the eq. (3.2) for F_{q^e+2} in order to be able to allow the opening of q^e on q^m by the addition to the action eq. (2.2) of the Chern-Simons term

$$\int A_{q^e+1} \wedge F_{q^e+2} \wedge F_{D-2q^e-3}\,, \tag{3.3}$$

and of a standard kinetic energy term for the new[6] field strength F_{D-2q^e-3}. Its dilaton coupling is chosen such that the intersection rules eq. (2.11) give integer intersection dimension between the new extremal electric $(D-2q^e-5)$-brane and its dual magnetic $(2q^e+1)$-brane. Charge conservation now reads

$$d * F_{q^e+2} = F_{q^e+2} \wedge F_{D-2q^e-3} + Q^e \delta_{D-q^e-1} \, . \tag{3.4}$$

Here and in what follows wedge products are defined up to signs and numerical factors. In the r.h.s. of eq. (3.4) the first term comes from the variation of the Chern-Simons term and the second one is the q^e-brane charge density. δ_{D-q^e-1} is the Dirac delta function in the directions transverse to the q^e-brane. We introduce here an explicit source term for the electric brane since, to study its opening, we want to extend to the branes themselves the validity of the usual closed brane solution . Such term is required because the equations of motion from which the intersecting brane solutions were derived do not contain any source term and are therefore valid only outside the sources.

Now it is easy to see that opening is consistent with charge conservation. Taking into account that in the configuration considered no brane is associated to the new field, we have

$$\begin{aligned} F_{D-2q^e-3} &= dA_{D-2q^e-4} \, , \\ dF_{q^e+2} &= Q^m \delta_{q^e+3} \, , \end{aligned} \tag{3.5}$$

and one can rearrange eq. (3.4) in the following way,

$$d(*F_{q^e+2} - F_{q^e+2} \wedge A_{D-2q^e-4}) = Q^e \delta_{D-q^e-1} - Q^m \delta_{q^e+3} \wedge A_{D-2q^e-4} \, . \tag{3.6}$$

A S^{D-q^e-1} sphere, which has only one point in common with the open q^e-brane, intersects the q^m-brane on a S^{D-2q^e-4} sphere surrounding the intersection. Integrating eq. (3.6) over the S^{D-q^e-1} sphere, one gets

$$0 = Q^e - Q^m \int_{S^{D-2q^e-4}} A_{D-2q^e-4} \, . \tag{3.7}$$

This equation can be rewritten as:

$$Q^e = Q^m . Q_1 \qquad \text{with} \qquad Q_1 \equiv \int_{S^{D-2q^e-4}} A_{D-2q^e-4} \, . \tag{3.8}$$

We see that, up to a closed form da_{D-2q^e-3} on the closed q^m-brane, the pull-back $\hat{A}^{(q^m)}_{D-2q^e-4}$ of the potential A_{D-2q^e-4} to this brane behaves like a $D-2q^e-4$-form field strength, magnetically coupled to the boundary. We thus write the field strength on the q^m-brane as

$$\mathcal{G}_{D-2q^e-4} = \hat{A}^{(q^m)}_{D-2q^e-4} - da_{D-2q^e-3} \, . \tag{3.9}$$

[6]If $D = 3q^e + 5$ there is no need to introduce a new field strength as the Chern-Simons terms can be build with the field strength F_{q^e+2}.

Equivalently, one may formulate the world-volume theory of the closed 'host' brane in terms of the world-volume Hodge dual $\mathcal{F}_{q^e+1} \stackrel{def}{=} {}^\star\mathcal{G}_{D-2q^e-4}$, thus interpreting the charge Q_1 at the end of the q^e-brane as an electric charge on the q^m-brane.

The above argument applied to the electric brane source of the new field F_{D-2q^e-3} shows that the new extremal electric brane can also be opened on the host closed magnetic brane q^m, provided the intersection rules are satisfied between the new and the original branes, as will indeed be the case for the problem studied in section 5. Its end carries a magnetic charge Q_2 which is the source of a field $\mathcal{H}_{q^e+1}$ on the brane which may tentatively be identified[7] with $\mathcal{F}_{q^e+1} \stackrel{\mathrm{def}}{=} {}^\star\mathcal{G}_{D-2q^e-4}$.

The world-volume point of view gives a picturesque way to determine which branes have to be added in order to have a consistent opening. Indeed, when a brane opens on a closed 'host' brane, the boundary appears from the world-volume point of view as a charged object under a world-volume field strength living in the closed brane. The world-volume Hodge dual of this object is the boundary of an other brane which can also be consistently opened on the same closed 'host' brane. The field strength associated to this new brane is precisely the one appearing in the Chern-Simons ensuring the consistency of the opening of the brane we started with.

In summary, having a theory of type eq. (2.2) in which some zero binding energy configurations give potential boundaries, it is possible to complete it in a well-defined way (adding Chern-Simons terms,[8] and when necessary new form field strengths, hence new branes) in order to ensure consistency of brane opening with charge conservation. In section 5, such dynamical requirement will be at work to reconstruct theories characterised by a coset symmetry, when dimensionally reduced, purely from brane considerations.

4. $D = 3$ cosets and intersection rules

It has been realised long ago that the scalars occurring in supergravity theories belong to cosets or non-linear realisations of a Lie group. This is a consequence of supersymmetry, but the Lie algebras that describe the cosets were not anticipated. They have been the subject of much study, and some classic examples are given in [22]. The more scalars one has, the bigger the corresponding Lie algebra. Consequently, when one performs a dimensional reduction of a given supergravity theory on tori, the algebras of the non-linear realisation grow in size corresponding to growth in the number of scalars in the dimensional reduction process. The full supergravity theories obtained in this way always admit a non-compact Lie group symmetry $\mathcal{G}$ which is manifest in the scalar sector of the theory. This

[7]Strictly speaking, this identification is not necessary for what follows but appears natural as it avoids a doubling of fields degrees of freedom on the brane q^m. In the M-theory case, this identification can be proven [7] and we surmise that it is a general consequence of the symmetries uncovered in this paper. Note that this identification implies that the field $f = da$ in eq. (3.9) *must* be present on the world volume of the q^m-brane to ensure gauge invariance [19] for the open branes.

[8]The coefficients of the Chern-Simons terms eq. (3.3) are not fixed in this qualitative discussion. Their precise values are important when one will discuss the potential symmetries. In the framework of M-theory they are usually fixed by supersymmetry. Nevertheless it is possible to fix them or at least quantise them generically using only consistency arguments without appealing to supersymmetry, see for instance [20, 21].

sector is often described by a non-linear realisation of the group in the form of a coset space $\mathcal{G}/\mathcal{H}$ where $\mathcal{H}$ is the maximal compact subgroup of $\mathcal{G}$. The finite dimensional Lie group $\mathcal{G}$ reaches its largest rank when the dimensional reduction is performed all the way down to three dimensions, since all the original fields can then indeed be expressed in terms of scalars. The complete lagrangian is then a conventional non-linear realisation of scalar fields.

For each finite dimensional semi-simple Lie group $\mathcal{G}$ one can consider the corresponding three dimensional scalar coset and in reference [8] a higher dimensional theory which leads upon dimensional reduction to the three dimensional scalar coset theory is found for each $\mathcal{G}$. This process is called oxidation and the theory in the highest dimension which leads to the three dimensional scalar coset for a given $\mathcal{G}$ is referred to as a maximally oxidised theory. In fact, the dimensional reduction of a generic field theory leads to a theory with no Lie algebra symmetry. However, there are examples of theories that possess no supersymmetry, but whose dimensional reduction leads to scalars that belong to non-linear realisations. Perhaps the best known is the dimensional reduction of pure gravity in any dimension. The occurrence and the uniqueness of a symmetry resulting from dimensional reduction were investigated in reference [11] and one can examine in detail how the resulting Lie group symmetry in three space-time dimensions places relations between the field content and couplings of the higher dimensional theory.

In this section we will perform the dimensional reduction of a generic theory described by actions of the type eq. (2.2), with the option to add some Chern-Simons terms when necessary. We will discuss the possible emergence of a coset symmetry in three dimensions. Some particular cases presented here have already been considered in [11] whose line of argument we follow. We will relate the possible emergence of symmetry to the intersection rules and thus to the possible existence of BPS configurations between extremal branes characterising a theory. We will uncover a precise relation between the intersection rules and the onset of a symmetry.

To fix notations in a self-contained presentation, we start by recalling the well-known dimensional reduction method. We will follow closely the discussion outlined in references [23] and [11].

Starting with a theory defined in D dimensions by an action of the type eq. (2.2), we compactify to $D-1$ dimensions and, remaining in the Einstein frame with the standard 1/2 kinetic term normalisation for the new scalar. This procedure amounts to take as compactification ansatz[9]

$$ds_D^2 = e^{2\beta_{D-1}\phi_2} ds_{D-1}^2 + e^{-2(D-3)\beta_{D-1}\phi_2}(dx^{D-1} + A_\mu dx^\mu)^2 \,. \tag{4.1}$$

We used the notation ϕ_2 for the scalar appearing in the first step of the dimensional reduction and we rename ϕ_1 the dilaton ϕ already present in the uncompactified theory defined by eq. (2.2). The compactified coordinate is x^{D-1}, $\mu = 0 \dots D-2$, and

$$\beta_D = \sqrt{\frac{1}{2(D-1)(D-2)}} \,. \tag{4.2}$$

[9] This corrects a typo in eq. (2.1) of ref. [11].

The gravitation part of the action eq. (2.2) becomes

$$\int d^D x \sqrt{-g_D}\, R_D = \int d^{D-1}x \sqrt{-g_{D-1}} \Bigg(R_{D-1} - \frac{1}{2}\partial_\mu \phi_2 \partial^\mu \phi_2 - \\ - \frac{1}{4} e^{-2(D-2)\beta_{D-1}\phi_2}\, F_{\mu\nu}F^{\mu\nu} \Bigg), \qquad (4.3)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

For each n_I-form field strength F_{n_I} in eq. (2.2) we get after reduction

$$\int d^D x \sqrt{-g_D} \frac{1}{2n_I!} e^{a_I \phi_1} F^2_{n_I} = \int d^{D-1}x \sqrt{-g_{D-1}} \Bigg(\frac{1}{2n_I!} e^{a_I\phi_1 - 2(n_I-1)\beta_{D-1}\phi_2} F'^2_{n_I} + \frac{1}{2(n_I-1)!} \times \\ \times e^{a_I\phi_1 + 2(D-1-n_I)\beta_{D-1}\phi_2}\, F^2_{n_I-1} \Bigg), \quad (4.4)$$

where

$$F'_{\mu_1\ldots\mu_n} = F_{\mu_1\ldots\mu_n} - n\, F_{[\mu_1\ldots\mu_{n-1}} A_{\mu_n]} \,. \qquad (4.5)$$

We can repeat this procedure step by step to obtain the theory on a p-torus. One has then obviously p scalars ϕ_j with $j = 2 \ldots p+1$ parametrising the radii of the torus, coming from the diagonal components of the metric in the compact dimensions. Additional scalars denoted $\chi_{\vec\alpha}$ arise from several origins. They come from potentials A^k_μ which arise when reducing gravity from $D+1-k$ to $D-k$ (see eqs. (4.1) and (4.3)) and also from the potentials associated to the F_{n_I} (when $p \geq n_I - 1$) with indices in the compact dimensions. In addition the n-form field strengths give additional scalars when $p = D - n - 1$ by dualising them. In particular when we reach $D = 3$ the $F^k_{\mu\nu}$ (with $k = 1 \ldots D-3$) coming from the gravity part of the action (i.e. the graviphotons) can be dualised to scalars, and we are left with only scalars. The action takes then the form

$$S = \int d^3x\, \sqrt{-g_3}\, \left(R_3 - \frac{1}{2}\partial_\mu \vec\phi \cdot \partial^\mu \vec\phi - \frac{1}{2}\sum_{\vec\alpha} e^{\sqrt{2}\vec\alpha\cdot\vec\phi} \partial_\mu \chi_{\vec\alpha} \partial^\mu \chi_{\vec\alpha} + \cdots \right), \qquad (4.6)$$

where $\vec\phi = (\phi_1, \ldots, \phi_{D-2})$, the $\vec\alpha$ are constant $(D-2)$-vectors[10] characterising each $\chi_{\vec\alpha}$. If we start in D dimensions without dilaton the vectors are of course $(D-3)$-dimensional as ϕ_1 is absent in that case. The ellipsis in eq. (4.6) stands for terms of order higher than quadratic in the $\chi_{\vec\alpha}$ scalars. They come from the modification of the field strengths eq. (4.5) in the dimensional reduction process and also from possible Chern-Simons terms in the uncompactified theory.

We now recall in which circumstances a lagrangian eq. (4.6) can be identified with a non-linear realisation of $\mathcal{G}$ in a coset space $\mathcal{G}/\mathcal{H}$ [23]. More precisely, we consider here a split (maximally non-compact) group $\mathcal{G}$ with generators $\vec H$ and $E^{\vec\alpha}$ where the Cartan subalgebra is generated by $\vec H$ and

$$[\vec H, E^{\vec\alpha}] = \vec\alpha E^{\vec\alpha} \,. \qquad (4.7)$$

[10] The normalisation factor $\sqrt{2}$ has been chosen for convenience. It will eventually correspond in the simply laced case to the standard normalisation of the roots, namely $\vec\alpha \cdot \vec\alpha = 2$.

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Roots split into positive and negative ones. In what follows we call a root positive (negative) if its first non-vanishing component, as counted from the right, is positive (negative). The positive roots can be written as linear combinations, with non-negative integer coefficients, of the so-called simple roots. The Cartan matrix, which uniquely determines the algebra of $\mathcal{G}$, is defined in terms of the simple roots and is given by

$$A_{ij} = 2\frac{\vec{\alpha}_i \cdot \vec{\alpha}_j}{\vec{\alpha}_i \cdot \vec{\alpha}_i}\,. \tag{4.8}$$

The cartan involution $\tau : (E^{\vec{\alpha}}, E^{-\vec{\alpha}}, \vec{H}) \to -(E^{-\vec{\alpha}}, E^{\vec{\alpha}}, \vec{H})$ can be used to construct the maximal compact subgroup $\mathcal{H}$ as the subgroup invariant under the involution. We can then construct $\mathcal{G}/\mathcal{H}$ non-linear realisations. We write the coset representatives by exponentiating the Borel subalgebra generated by the Cartan and the positive root generators, namely,

$$\mathcal{U} = e^{\frac{1}{\sqrt{2}}\vec{\phi}\cdot\vec{H}} \sum_{\vec{\alpha}>0} e^{\chi_{\vec{\alpha}}\cdot E^{\vec{\alpha}}}\,, \tag{4.9}$$

where the sum is only over the positive roots. One can show that the scalar lagrangian

$$\mathcal{L}_{\mathcal{G}/\mathcal{H}} = \frac{1}{4}\mathrm{Tr}\left(\partial_\mu \mathcal{M}^{-1}\partial^\mu \mathcal{M}\right), \tag{4.10}$$

where

$$\mathcal{M} = \mathcal{U}^{\#}\mathcal{U}\,, \tag{4.11}$$

is invariant under global $\mathcal{G}$ transformations and local $\mathcal{H}$ transformations $\mathcal{U} \to h\mathcal{U}g$. Here we used the generalised transpose acting on the generators: $X^{\#} = \tau(X^{-1})$. If we normalise the cartan and positive-root generators so that

$$\mathrm{Tr}(H_i H_j) = \delta_{ij} \qquad \mathrm{Tr}(E^{\vec{\alpha}}E^{\vec{\beta}}) = 0 \qquad \mathrm{Tr}\left(E^{\vec{\alpha}}E^{-\vec{\beta}}\right) = \delta^{\vec{\alpha}\vec{\beta}}\,, \tag{4.12}$$

one can show that $\mathcal{L}_{\mathcal{G}/\mathcal{H}}$ is precisely the scalar part of the lagrangian in the action eq. (4.6).

Thus it follows that the action eq. (2.2) when dimensionally reduced to three dimensions has a $\mathcal{G}/\mathcal{H}$ symmetry *if* the vectors $\vec{\alpha}$ obtained from the compactification can be identified with the positive roots of a group $\mathcal{G}$ and *if*, when necessary, some precise Chern-Simons terms are added in the uncompactified theory. Adding some precise Chern-Simons terms in eq. (2.2) may be indeed required in order to match the terms obtained in the dimensional reduction, which are included in the ellipsis in eq. (4.6), with $\mathcal{L}_{\mathcal{G}/\mathcal{H}}$ (see for instance [23]). The requirement that the $\vec{\alpha}$ correspond to positive roots is thus a necessary condition to uncover a symmetry.

We are now ready to discuss the dimensional reduction of a generic theory described by the action eq. (2.2) and the necessary conditions for the emergence of a coset symmetry in three dimensions. The discussion generalises the one of ref.[11]. We will then uncover the relation between the onset of symmetry and the intersection rules.

We first recall the well-known dimensional reduction of pure gravity (see eq. (4.3)) down to three dimensions, which leads to $\mathcal{G} = \mathrm{SL}(D-2)$ whose algebra is A_{D-3}. The scalars corresponding to the simple roots of A_{D-3} are of two kinds.

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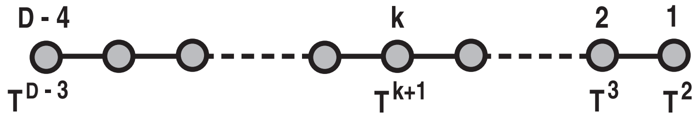

Figure 1: The gravity line. Dynkin diagram of A_{D-4} generated by the dimensional reduction to 3 dimensions.

There are first $D-4$ scalars which are the components A^k_{D-k-1} of the potentials coming from $g_{D-k,D-k-1}$, $k = 1 \dots D-4$. These are obtained by performing the "fastest" reduction on the potentials A^k_μ (see eq. (4.3)) to obtain a scalar going from $D-k$ to $D-k-1$ when compactifying on T^{k+1}. The corresponding simple roots $\vec{\alpha}^g_k$ are given by

$$\begin{aligned} \vec{\alpha}^g_k &= \sqrt{2}(0; \underbrace{0,\dots,0}_{k-1 \text{ terms}}, -(D-k-1)\beta_{D-k}, (D-k-3)\beta_{D-k-1}, \underbrace{0,\dots,0}_{D-4-k \text{ terms}}) , \\ k &= 1 \dots D-4 \, . \end{aligned} \tag{4.13}$$

They define a subalgebra A_{D-4}. We have indeed

$$\vec{\alpha}^g_k \cdot \vec{\alpha}^g_l = \begin{cases} 2 & k = l \\ -1 & |k-l| = 1 \\ 0 & |k-l| \geq 2 \end{cases} \tag{4.14}$$

The first component of $\vec{\alpha}^g_k$ associated to the dilaton ϕ in the original uncompactified theory eq. (2.2) is always zero. The corresponding Dynkin diagram with $D-4$ nodes, which from now on we will refer to as *the gravity line* is depicted in figure 1.

The remaining scalar corresponding to the missing simple root leading to the full A_{D-3} comes from dualising in three dimensions the first vector (graviphoton) that arises in the stepwise procedure namely the vector appearing already in $D-1$ dimensions. The corresponding simple root is

$$\vec{\alpha}^{gp} = \sqrt{2}(0; (D-2)\beta_{D-1}, \beta_{D-2}, \beta_{D-3}, \dots, \beta_3) \, . \tag{4.15}$$

Note that this simple root $\vec{\alpha}^{gp}$ has a non-vanishing scalar product with $\vec{\alpha}^g_1$ (i.e with the simple root already appearing when compactifying on T^2). One has indeed $\vec{\alpha}^g_k \cdot \vec{\alpha}^{gp} = -\delta_{k,1}$. Consequently it attaches itself to the right of the gravity line. The other $\frac{1}{2}(D-4)(D-3)$ scalars coming from the reduction of gravity down to three dimensions give all the positive roots of A_{D-3}.

We now turn to theories with forms given by eq. (2.2) and begin by considering a theory with only one n_A-form F_{n_A} with dilaton coupling a_A (and $n_A \leq D/2$). Let us consider the first scalar arising from the n_A-form upon dimensional reduction up to $p = n_A - 1$. The vector $\vec{\alpha}^e_{n_A}$ associated to this scalar[11] will from now on be called *the would-be electric root.*

[11] The other scalars obtained by further dimensional reduction give $\vec{\alpha}$-vectors that are linear combinations with positive integer coefficients of $\vec{\alpha}^e_{n_A}$ and of the $\vec{\alpha}^g_k$.

It is given by

$$\vec{\alpha}^e_{n_A} = \left(\frac{a_A}{\sqrt{2}}; \underbrace{b_{(n_A,D)}\,\beta_{D-1}, b_{(n_A,D)}\,\beta_{D-2}, \ldots, b_{(n_A,D)}\,\beta_{D-n_A+1}}_{n_A-1 \text{ terms}}, \underbrace{0,\ldots,0}_{D-n_A-2 \text{ terms}} \right), \tag{4.16}$$

with

$$b_{(n_A,D)} = \sqrt{2}\,(D - n_A - 1)\,. \tag{4.17}$$

First, we compute the scalar product of the would-be electric root with the gravity line.[12] Using eq. (4.13) and eq. (4.16) we find, *for any D and dilaton coupling a_A*,

$$\vec{\alpha}^g_k \cdot \vec{\alpha}^e_{n_A} = -\delta_{k,n_A-1}\,. \tag{4.18}$$

We then evaluate the length of the would-be electric root using eq. (4.16). We find

$$\begin{aligned} \vec{\alpha}^e_{n_A} \cdot \vec{\alpha}^e_{n_A} &= \frac{(n_A-1)(D-n_A-1)}{(D-2)} + \frac{a_A^2}{2}\,, \\ &= \frac{(q^e_A+1)(q^m_A+1)}{(D-2)} + \frac{a_A^2}{2}\,, \\ &= \bar{q}^{(e_A,m_A)} + 1\,. \end{aligned} \tag{4.19}$$

We thus see that the square length of the would-be electric root associated to F_{n_A} can be written in terms of the intersection rule equation eq. (2.11) giving the intersection between the electric and the magnetic brane charged under F_{n_A}.

From now on we will restrict ourselves to simply laced groups. In our normalisation all their roots are of square length two (see footnote 10). In order for the would-be root to be a root, one must have $\vec{\alpha}^e_{n_A} \cdot \vec{\alpha}^e_{n_A} = 2$. Consequently *the existence of a BPS configuration in the original theory, consisting of an electric extremal p-brane ($p \geq 1$) and its magnetic dual whose intersection is $\bar{q}^{(e_A,m_A)} = 1$, is a necessary condition to have after dimensional reduction an enhanced simply laced Lie group symmetry.*

Let us choose the dilaton coupling to F_{n_A} such that $\bar{q}^{(e_A,m_A)} = 1$. Using the scalar product eq. (4.18) of the would-be electric root with the gravity line, we can draw a would-be Dynkin diagram where the would-be electric root associated to F_{n_A} is connected to the $(n_A-1)^{th}$ node of the gravity line as depicted in figure 2.

In the dimensional reduction, there is an additional scalar coming from the F_{n_A} which arises when one reaches n_A+1 non-compact dimensions. It is obtained by dualising the n_A-form to a scalar. The corresponding would-be magnetic root $\vec{\alpha}^m_{n_A}$ is given by

$$\vec{\alpha}^m_{n_A} = \left(-\frac{a_A}{\sqrt{2}}; \underbrace{c_{n_A}\,\beta_{D-1}, c_{n_A}\,\beta_{D-2}, \ldots, c_{n_A}\,\beta_{n_A+1}}_{D-n_A-1 \text{ terms}}, \underbrace{0,\ldots,0}_{n_A-2 \text{ terms}} \right), \tag{4.20}$$

with

$$c_{n_A} = \sqrt{2}\,(n_A - 1)\,. \tag{4.21}$$

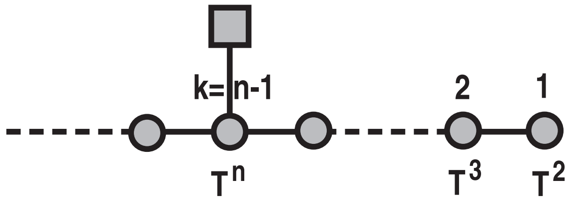

Figure 2: Would-be Dynkin diagram. The would-be electric root associated to F_n is represented by a shaded square.

The scalar product of the would-be magnetic root with the gravity line is given by $\vec{\alpha}^g_k \cdot \vec{\alpha}^m_{n_A} = -\delta_{k,D-n_A-1}$ for any D and a_A. The square length of $\vec{\alpha}^m_{n_A}$ gives again the intersection rule equations between the electric and the magnetic brane namely $\vec{\alpha}^m_{n_A} \cdot \vec{\alpha}^m_{n_A} = \bar{q}^{(e_A,m_A)}+1$. Consequently a necessary condition for the would-be magnetic root to be a root is the same as previously. This root may not be simple. We note that the scalar product of the would-be magnetic root with the would-be electric root gives the intersection rule equation between two *electric* branes

$$\begin{aligned}
\vec{\alpha}^e_{n_A} \cdot \vec{\alpha}^m_{n_A} &= \frac{(n_A-1)^2}{(D-2)} - \frac{a_A^2}{2}\,, \\
&= \frac{(q^e_A+1)(q^e_A+1)}{(D-2)} - \frac{a_A^2}{2}\,, \\
&= \bar{q}^{(e_A,e_A)} + 1\,,
\end{aligned} \tag{4.22}$$

and hence, from eq. (2.12), is integer if $\bar{q}^{(e_A,m_A)} = 1$.

If we now generalise the previous discussion to a theory with more than one form F_{n_A}, we again find relations between the intersection rules and the would-be roots. For instance considering a F_{n_A} form and a F_{n_B} form with $n_B > n_A$ the scalar product of the two corresponding would-be electric roots is given by

$$\begin{aligned}
\vec{\alpha}^e_{n_A} \cdot \vec{\alpha}^e_{n_B} &= \frac{(D-q^e_B-3)(q^e_A+1)}{(D-2)} + \frac{a_A a_B}{2}\,, \\
&= \bar{q}^{(e_A,m_B)} + 1\,,
\end{aligned} \tag{4.23}$$

which is the intersection rule between the magnetic extremal brane charged under F_{n_B} and the electric extremal brane charged under F_{n_A}.

In this section we have derived important relations between the existence of BPS configurations of intersecting branes and the presence of a potential underlying symmetry of a given theory. In particular, we have shown that the existence of BPS intersecting solutions (with one common direction) between $e-m$ pairs of extremal branes ($p \geq 1$) for each form field strength present in a theory given by eq. (2.2) is a necessary condition to have an enhanced symmetry corresponding to a simply laced Lie algebra when the theory is dimensionally reduced.

[12] The scalar product of the would-be electric root with the graviphoton is one for any D and a_A. This implies that when forms are present in a theory with symmetry, the graviphoton is never a simple root. Consequently we focus on the gravity line.

We briefly comment on the non-simply laced groups. These algebras have long and short roots. If we normalise the long roots to have square length two then the short roots will have integer square length for all the non-simply laced Lie algebra except one:[13] G_2 where the short root has square length 2/3. In order for a theory given by eq. (2.2) to give a non-simply laced symmetry B_n, C_n or F_4 (except G_2) one must have, as a necessary condition, intersecting BPS configurations with $\bar{q} = 1$ or 0 for all the $e - m$ pairs of branes of the theory.

5. Brane dynamics and symmetry

In this section, using the result of the previous section and the opening of branes described in section 3, we will be able to reconstruct all the oxidation end points described in reference [8] leading in three dimensions to all (split) simply laced group[14] $\mathcal{G}$.

Our starting point will be a theory given by eq. (2.2) in D dimensions with only one n_A-form field strength F_{n_A} and its dilaton coupling a_A. We will fix the dilaton coupling such that there exists a zero-binding energy configuration between the electric q^e_A-brane ($q^e_A = n_A - 2$) and the magnetic q^m_A-brane with $\bar{q}^{(e_A,m_A)} = 1$. As explained in the previous section this is a necessary condition in order to find a new symmetry. Once the dilaton coupling of the form is fixed, we require that, when the dimensionality of an intersection permits opening, the latter is consistent with charge conservation. Namely we will impose that, if $\bar{q}^{(e_A,m_A)} = q^e_A - 1$, the electric brane open on the magnetic brane. As explained in section 3, this requires the introduction of a specific Chern-Simons term in the action, which may contain a new form field strength F_{n_B}. The dilaton coupling a_B of the new form field strength is then again fixed modulo its sign by the necessary condition $\bar{q}^{(e_B,m_B)} = 1$. The intersection rules between the branes corresponding to the different forms can be calculated. We can then check if new openings are possible. If it is the case, we iterate the procedure until consistency of all the openings are ensured. In this way, we will be able to reconstruct all the maximally oxidised theories corresponding to the simply laced groups. This leads to the following conclusion: *The existence of BPS configurations with $\bar{q} = 1$ between any electric extremal p-brane ($p \geq 1$) and its magnetic dual, along with the requirement of consistency of brane opening in the original uncompactified theory (characterised by at most one dilaton), is a necessary and sufficient condition to have a theory whose dimensional reduction down to three dimensions has a simple simply laced group $\mathcal{G}$ symmetry.*

We now turn to the proof. We start with a theory in D dimensions ($D \geq 3$) with one n-form field strength F_n and dilaton coupling a_n. The dimension of the electric brane corresponding to F_n is $q = n - 2 \geq 1$ with $2q + 4 \leq D$ (by convention the electric brane is always smaller than the magnetic one). Requiring $\bar{q}^{(e,m)} = 1$ and using eq. (4.19) we get, up to a sign, the dilaton coupling

$$a_n^2(D) = 2\frac{(1-q)D + (q^2 + 4q - 1)}{D - 2}\,. \tag{5.1}$$

[13] We thank Marc Henneaux for a discussion on the specificity of G_2.

[14] The A-series corresponds to pure gravity (without forms), which is the basic ingredient of our construction.

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We must have

$$a_n^2 \geq 0\,. \tag{5.2}$$

For $q = 1$, the condition eq. (5.2) is satisfied for *any* dimension D. The value of the dilaton coupling is $a_3^2 = 8/(D-2)$. Thus we recover eq. (2.14). We get the model eq. (2.13) characterised by an electric $1F$ and a magnetic $NS(D-5)$ with intersections given by eq. (2.15). The BPS configuration between the $e-m$ pair is given by a $1F$ *living inside* the $NS(D-5)$. No opening can occur, henceforth there is no need to add new form or Chern-Simons terms. The theory given by the action eq. (2.13) should then lead after dimensional reduction down to three to a coset model with $\mathcal{G}$ simply laced. This is the case. Indeed, the would-be electric root and the would-be magnetic root associated to H_3 are simple roots and we find the D_n-series with $D = n+2$.

For $q > 1$ eq. (5.2) leads to the following constraint on D,

$$2q + 4 \leq D \leq q + 5 + \frac{4}{q-1}\,. \tag{5.3}$$

The constraint eq. (5.3) can only be satisfied for $q = 2$ and $q = 3$. We have 5 possibilities

$$\begin{aligned} q = 2 \qquad &\text{and} \begin{cases} D = 11 \\ D = 10 \\ D = 9 \\ D = 8 \end{cases} \\ q = 3 \qquad &\text{and } D = 10\,. \end{aligned} \tag{5.4}$$

For $q = 2$ (F_4) the electric brane can always potentially open on the magnetic dual.

$q = 2$, $D = 11$. In this case eq. (5.1) gives $a_4 = 0$, namely no dilaton coupling. There is a 2-brane and its dual magnetic 5-brane with the intersection $5 \cap 2 = 1$. To open the 2-brane on the 5-brane, it is necessary to add a Chern-Simons term. Here we are in the special case $D = 3q + 5$ and there is no need to add a new field strength to build the Chern-Simons term. This term is proportional to $F_4 \wedge F_4 \wedge A_3$. Our construction procedure stops here as no new form are introduced. This theory is expected, from section 4, to have a symmetry E_8 after dimensional reduction to three dimensions. This is the case. The intersection of the 2-brane with the 5-brane along one common dimension, together with the consistency condition on openings, has fixed the action to be that of eleven dimensional supergravity whose enhanced symmetry in the dimensional reduction down to three is indeed E_8 [24].

$q = 2$, $D = 10$. In this case, eq. (5.1) gives[15] $a_4 = 1/2$. There is a 2-brane that we denote, anticipating the result, $D2$, and its magnetic dual which is a 4-brane, $D4$. According to our logic, we demand that the opening of the $D2$ on the $D4$ be consistent. This requires the introduction of a Chern-Simons term proportional to $A_3 \wedge F_4 \wedge H_3$ where H_3 is a new field strength, giving a new electric 1-brane (that we call $1F$), with its associated dilaton coupling a_3. From eq. (5.1) one finds $a_3 = -1$. The minus sign is imposed by the requirement

[15] The plus sign here is just a matter of convention only the relative signs between dilaton couplings matters.

that the intersection rules between the $1F$ and the $D4$ gives a BPS configuration, namely $1F \cap D4 = 0$. The $1F$ can potentially open on the $D2$ in virtue of the intersection rule $1F \cap D2 = 0$. Consistency of this opening implies the introduction of a field strength F_2 through a Chern-Simons type term in the action leading to $d * H_3 \propto *F_4 \wedge F_2$. Note that this is an opening of an electric brane on another electric brane. The branes corresponding to this new F_2 are a 0-brane ($D0$) and a 6-brane ($D6$). The corresponding dilaton coupling a_2^2 is again fixed by eq. (5.1) and the sign is fixed by requiring that the intersection rule for $1F$ and $D0$ yields a BPS configuration, namely $1F \cap D0 = 0$. One gets $a_2 = 3/2$. Having fixed all the dilaton couplings, it is then possible to check all the intersection rules and to introduce a Chern-Simons like term for each possible opening. In this way one checks that no other forms are required and one recovers type IIA theory.

In our constructive approach, one thus recovers a theory having after reduction down to three dimensions a symmetry which is again, as expected, E_8. Being the dimensional reduction of 11 dimensional supergravity, it is not an oxidation endpoint.

$q = 2$, $D = 9$. Eq. (5.1) now gives $a_4 = 2/\sqrt{7}$. There is an electric 2-brane (noted $\mathbf{2}$) and its dual magnetic 3-brane (noted $\mathbf{3}$) with the following pairwise BPS configurations read off, from eq. (2.11),

$$\mathbf{2} \cap \mathbf{2} = 0 \qquad \mathbf{3} \cap \mathbf{3} = 1 \qquad \mathbf{2} \cap \mathbf{3} = 1\,. \tag{5.5}$$

The opening of $\mathbf{2}$ onto the 'host' $\mathbf{3}$ requires the introduction a new form F_2 with $a_2^2 = 16/7$ and a Chern-Simons term in the action proportional to

$$F_4 \wedge A_3 \wedge F_2 \qquad \text{with} \qquad F_4 = dA_3\,. \tag{5.6}$$

One gets a new electric 0-brane (noted $\mathbf{0}$) and a new magnetic 5-brane (noted $\mathbf{5}$). There is one BPS-configuration possible associated to this form, namely $\mathbf{5} \cap \mathbf{5} = 3$. The sign of a_2 is chosen to allow new BPS configurations between branes charged under different forms. Picking $a_2 = -4/\sqrt{7}$ one gets the following additional BPS intersections,

$$\mathbf{2} \cap \mathbf{0} = 0 \qquad \mathbf{3} \cap \mathbf{0} = -1 \qquad \mathbf{5} \cap \mathbf{3} = 3\,, \tag{5.7}$$

$$\mathbf{2} \cap \mathbf{5} = 1\,. \tag{5.8}$$

The last intersection eq. (5.8) yields the possible opening of $\mathbf{2} \to \mathbf{5}$ whose consistency is precisely ensured by the Chern-Simons term already introduced in eq. (5.6). There is no more possible opening hence our constructive procedure stops here. This theory is expected, from section 4, to have a symmetry E_7 after dimensional reduction to three dimensions. This is indeed the case! It corresponds to the theory which is the oxidation endpoint of $\mathcal{G} = E_7$ [8].

$q = 2$, $D = 8$. The dilaton coupling a_4 is, from eq. (5.1), $a_4 = -1$ (the minus sign is purely conventional). There is an electric 2-brane $\mathbf{2_e}$ and its magnetic dual which is also a 2-brane, $\mathbf{2_m}$. One has the following BPS-configurations,

$$\mathbf{2_e} \cap \mathbf{2_e} = 0 \qquad \mathbf{2_m} \cap \mathbf{2_m} = 0 \qquad \mathbf{2_e} \cap \mathbf{2_m} = 1\,. \tag{5.9}$$

The opening of $\mathbf{2_e} \to \mathbf{2_m}$ requires the introduction of a Chern-Simons terms in the action of the following form,

$$A_3 \wedge F_4 \wedge F_1 \qquad \text{with} \qquad F_4 = dA_3\,, \tag{5.10}$$

hence the introduction of a new one-form F_1 and the corresponding (-1)-electric brane, $\mathbf{-1}$, and its dual magnetic 5-brane, $\mathbf{5}$. The dilaton coupling is, from eq. (5.1), $a_1^2 = 4$. Its sign is fixed to be plus by the requirement that there exists a further BPS-configuration whose opening is also consistent with the Chern-Simons term eq. (5.10), namely,

$$\mathbf{2_e} \cap \mathbf{5} = 1\,. \tag{5.11}$$

With the introduction of F_1, and a_1 fixed to be 2, there are no further possible openings and our building procedure stop here. This theory is expected, from section 4, to have a symmetry E_6 after dimensional reduction to three dimensions. This is indeed the case! It corresponds to the theory which is the oxidation endpoint of $\mathcal{G} = E_6$.

$q = 3$, $D = 10$. One has a five form field strength F_5 and from eq. (5.1), $a_5 = 0$, namely no dilaton coupling. In order to have a symmetry and avoid a degeneracy in the brane spectrum we impose a self-duality condition on F_5. One has a 3-brane ($\mathbf{3}$) and the following BPS configuration,

$$\mathbf{3} \cap \mathbf{3} = 1\,. \tag{5.12}$$

There is no possible opening, thus no need to add new forms. This theory in $D = 10$ dimensions with a self-dual 5-form gives indeed, again after dimensional reduction down to three, $\mathcal{G} = E_7$ [8]. It constitutes an oxidation endpoint if we allow for the self-duality condition.

6. Intersecting branes in $\mathcal{G}^{+++}$

In this section we analyse the moduli space of the intersecting brane solutions of the theories discussed above, which are the oxidation endpoints for the simple simply laced Lie groups $\mathcal{G}$.

Recall that these solutions were obtained by enforcing the ansatz eq. (2.7) whose significance we now discuss. We rewrite this equation in terms of the moduli

$$\begin{aligned} p^1 &= \ln B \\ p^a &= \ln G \qquad\qquad a = 2, \ldots, D-p \\ p^{D-p+i} &= \ln C_i \qquad\qquad i = 1, \ldots, p \end{aligned} \tag{6.1}$$

and we get

$$(p+3-D)\, p^a = p^1 + \sum_{i=1}^{p} p^{D-p+i} \qquad a = 2, \ldots, D-p\,. \tag{6.2}$$

For the extremal intersecting brane solutions eqs. (2.8) and (2.9) one has

$$p^1 = -\sum_A \frac{D - q_A - 3}{\Delta_A} \ln H_A\,,$$

$$\begin{aligned} p^a &= \sum_A \frac{q_A+1}{\Delta_A} \ln H_A \,, \\ p^{D-p+i} &= -\sum_A \frac{\delta^{(i)}_A}{\Delta_A} \ln H_A \,, \\ \phi &= \sum_A \frac{D-2}{\Delta_A} \varepsilon_A a_A \ln H_A \,. \end{aligned} \tag{6.3}$$

For simply laced groups, $\Delta_A/(D-2) = 2$. Taking into account the intersection rule equation eq. (2.11) and the ansatz eq. (6.2), one gets

$$\sum_{\alpha=1}^{D} (p^\alpha)^2 - \frac{1}{2}(\sum_{\alpha=1}^{D} p^\alpha)^2 + \frac{1}{2}\phi^2 = \frac{1}{2}\sum_A \ln^2 H_A(x^a)\,. \tag{6.4}$$

The relations eqs. (6.2) and (6.4) have a group-theoretical significance which we shall uncover. This section is based on reference [13] to which the reader is referred for more detailed discussions.

We first recall that the Lie algebra $\mathcal{G}$ can be embedded in a very extended Kac-Moody algebra $\mathcal{G}^{+++}$. The simple roots of $\mathcal{G}^{+++}$ are given by adding two nodes to the gravity line of the Dynkin diagram of the affine extension $\mathcal{G}^{+}$ of $\mathcal{G}$, thus increasing by three the rank of $\mathcal{G}$. The resulting Dynkin diagrams for simply laced algebras are shown in figure 3.

The group $\mathrm{SL}(D)$ defined by this triple extended gravity line can be extended to the full deformation group $\mathrm{GL}(D)$ whose algebra, generated by D^2 generators $K^\alpha{}_\beta$, $\alpha, \beta = 1, \ldots, D$, is a subalgebra of $\mathcal{G}^{+++}$. The $K^\alpha{}_\beta$ satisfy the following commutation relations

$$[K^\alpha{}_\beta, K^\gamma{}_\delta] = \delta^\gamma{}_\beta K^\alpha{}_\delta - \delta^\alpha{}_\delta K^\gamma{}_\beta \,. \tag{6.5}$$

One considers the Cartan subalgebra of $\mathcal{G}^{+++}$ generated by the $K^\alpha{}_\alpha$ and $q = r - D$ abelian generators R_u where r is the rank of $\mathcal{G}^{+++}$. We write the corresponding abelian group element g as

$$g = \exp(\sum_{\alpha=1}^{D} p^\alpha K^\alpha{}_\alpha) \exp(\sum_{u=1}^{q} \phi^u R_u)\,. \tag{6.6}$$

The group parameters $\{p^\alpha, \phi^u\}$ are related, for diagonal metric field configurations, to the metric fields $g_{\alpha\alpha}$ by

$$g_{\alpha\alpha} = e^{2p^\alpha} \eta_{\alpha\alpha}\,, \tag{6.7}$$

where $\eta_{\alpha\alpha}$ is the minkowskian metric $(-, +, +, \ldots, +)$. The ϕ^u are identified to dilaton fields. For the simply laced groups considered here there is at most one dilaton. It was shown in reference [13] that compactification of this theory on a k-torus in a flat background could be interpreted as the embedding of the algebra $\mathcal{G}^{(k)}$ into $\mathcal{G}^{+++}$ obtained by deleting, starting from the left, $D-k$ nodes of its gravity line together with nodes attached to the deleted ones. Rewriting the group element eq. (6.6) in the Chevalley basis of $\mathcal{G}^{+++}$ as $g = e^{q^m H_m}$, it amounts to set $q^m = 0$ for $m = 1, \ldots, D-k$ along with q^x where x labels

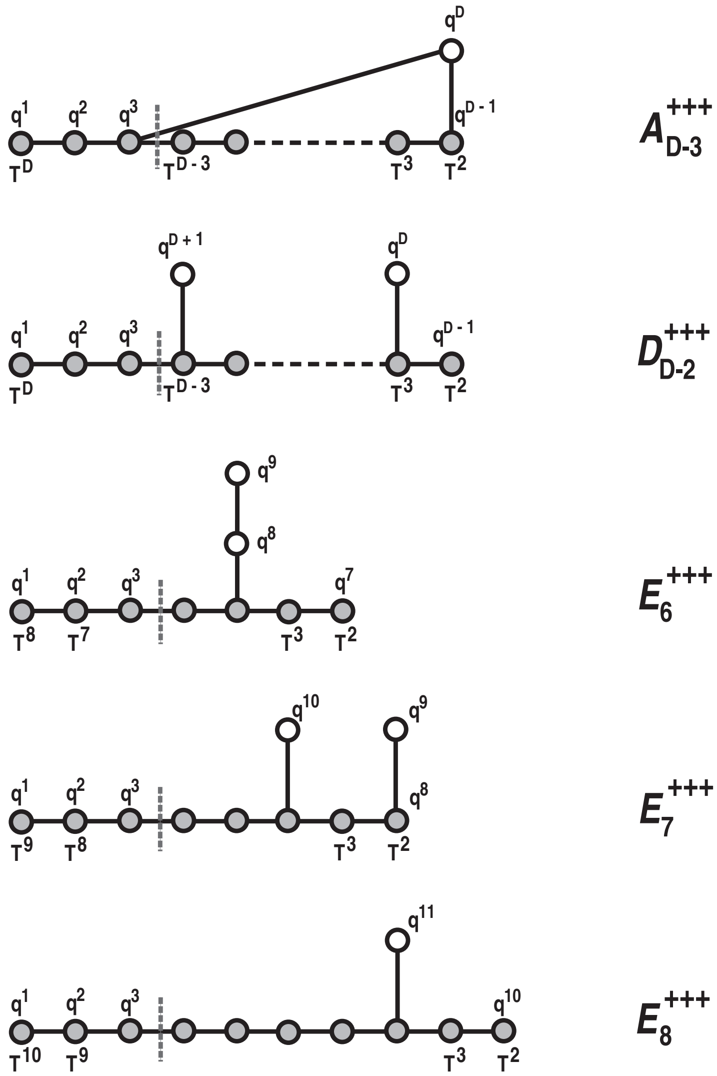

Figure 3: Dynkin diagram of simply laced Kac-Moody algebras $\mathcal{G}^{+++}$. The nodes of the gravity line are shaded. The nodes characterised by the Chevalley parameters q^1, q^2, q^3 are the Kac-Moody extensions of the Lie algebras $\mathcal{G}$ whose Dynkin diagrams are depicted by the nodes on the right of the vertical dashed lines. The triple extension A^{+++}_{D-3}, stemming from pure gravity, is included for sake of generality.

a possible node attached to one of the q^m (see Fig.3). This embedding is characterised by eq. (3.29) of reference [13] which can be written as

$$(k+2-D)\, p^a = \sum_{i=1}^{k} p^{D-k+i}\,, \qquad a = 1, \ldots, D-k\,, \tag{6.8}$$

where the $p^\alpha = \{p^a, p^{D-k+i}\}$ are related to the background metric through eq. (6.7). For such an embedding the group $S(\mathcal{G}^{(k)})$ generated by the Weyl generators of $\mathcal{G}^{(k)}$ and by the outer automorphisms of its Dynkin diagram is a subgroup of the group $S(\mathcal{G}^{+++})$ which leaves invariant the quadratic form

$$\sum_{\alpha=1}^{D}(p^{\alpha})^2 - \frac{1}{2}\left(\sum_{\alpha=1}^{D} p^{\alpha}\right)^2 + \frac{1}{2}\phi^2 \,. \tag{6.9}$$

Taking $k = p+1$ in eq. (6.8) and interchanging the time modulus p^1 with p^{D-p} one recovers the ansatz eq. (6.2). The interchange of moduli is a Weyl transformation in $\mathcal{G}^{+++}$. Thus the ansatz eq. (6.2) defines an embedding $\mathcal{G}^{(p+1)}$ in $\mathcal{G}^{+++}$ conjugate by Weyl reflection to the one defined by eqs. (6.8) and (6.9), provided the right hand side of eq. (6.4) is constant under the group $S(\mathcal{G}^{(p+1)})$. This is ensured if time is compactified along with the original p dimensions. When there exists an underlying string theory associated to the $\mathcal{G}^{+++}$ theory and an interpretation of Weyl transformations in terms of T-dualities [13], it amounts to allow T-duality along the time direction [25]. One can show, using the formalism of reference [13] that the embedding eq. (6.2) expressed in the Chevalley basis corresponds to the conditions $q^1 = q^2 = \ldots = q^{D-p}$.

It is of interest to compare for a given maximally oxidised theory the embedding of $\mathcal{G}^{(p+1)}$ in $\mathcal{G}^{+++}$ describing the intersecting brane solutions with the embedding of the overextended Kac-Moody algebra $\mathcal{G}^{++}$ in $\mathcal{G}^{+++}$ that characterises the cosmological Kasner solutions. These can be written as

$$ds^2 = -e^{2p^1(t)}dt^2 + \sum_{\alpha=2}^{D} e^{2p^{\alpha}(t)}\ (dx^{\alpha})^2; p^{\alpha}(t) = \tilde{p}^{\alpha} t \quad \text{with} \quad \tilde{p}^{\alpha} = \text{constant}\,, \tag{6.10}$$

$$p^1 = \sum_{\alpha=2}^{D} p^{\alpha}\,, \tag{6.11}$$

together with eq. (6.4), with the right hand side put equal to zero [13]. The embedding of this overextended Kac-Moody algebra $\mathcal{G}^{++}$ is defined by $q^1 = 0$ or equivalently by eq. (6.11). Its physical relevance was put into evidence by its occurrence in the cosmological billiards describing the evolution of a universe in the vicinity of a cosmological singularity [26] for all maximally oxidised theories [27]. The embedding of $\mathcal{G}^{(p+1)}$ in $\mathcal{G}^{+++}$ is also an embedding in its subalgebra $\mathcal{G}^{++}$ defined by $q^1 = q^2$ which is conjugate to the one defined by $q^1 = 0$. Hence, although both the algebra spanned by intersecting branes solutions and by the cosmological billiard solutions are both subalgebra of the very extended Kac-Moody algebra $\mathcal{G}^{+++}$, the first one cannot be embedded in the second and the common origin of their symmetry is only revealed at the level of the very extended Kac-Moody algebra.

It is also possible to interpret the embedding eq. (6.8) in terms of extremal intersecting branes where time is transverse to all branes. Such intersecting brane solutions [28] indeed exist in exotic theories [29]. Their role in the context of full $\mathcal{G}^{+++}$ invariance will not be discussed here.

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